Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

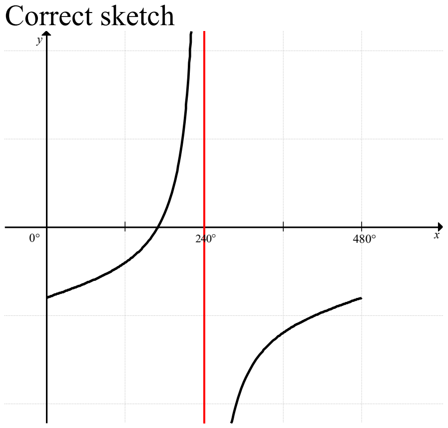
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$(x-3)^2-8$	**B2**	**B1** for $(x-3)^2+k$ where $k \neq -8$ or $a = -3$ or $(x+m)^2-8$ where $m \neq -3$ or $b = -8$
1(b)	$(3, -8)$	**B1**	**strict FT** *their* a and b
2	$m = \dfrac{9-5}{8-6}$ oe	**M1**	
	$9 =$ *their* $2(8) + c$ oe or $5 =$ *their* $2(6) + c$	**M1**	
	$\ln y = 2\ln x - 7$	**A1**	
	Correct completion to answer: $y = e^{\ln x^2 - 7} = e^{-7}x^2$ nfww	**A1**	
	Alternative $\ln y = p + q\ln x$ soi	 **(B1)**	
	$m = \dfrac{9-5}{8-6}$ oe	**(M1)**	
	$9 =$ *their* $2(8) + c$ oe or $5 =$ *their* $2(6) + c$	**(M1)**	
	$y = e^{-7}x^2$	**(A1)**	
3(a)	$4x - 1$ *9 oe and $4x - 1$*–9 oe OR $16x^2 - 8x - 80$*0 oe soi	**M1**	where * could be = or any inequality sign
	$x > \frac{5}{2}$, $x < -2$ only; **mark final answer**	**A2**	not from wrong working **A1** for CV $\frac{5}{2}$, –2 oe If M0 then **SC1** for any correct inequality with at most one extra inequality

Question	Answer	Marks	Partial Marks
3(b)	$\left(2\sqrt{x}-3\right)\left(\sqrt{x}-4\right)$ or $x = u^2$ and $(2u - 3)(u - 4)$ oe soi	**M1**	
	$\sqrt{x} = \frac{3}{2}$, $\sqrt{x} = 4$ oe	**A1**	
	$x = \frac{9}{4}$, $x = 16$	**B1**	**FT** their $\sqrt{x}$
	Alternative $(2x+12)^2 = \left(11x^{\frac{1}{2}}\right)^2$ simplified to $4x^2 - 73x + 144 = 0$	**(M1)**	
	solves 3 term quadratic in x	**(M1)**	
	$x = \frac{9}{4}$, $x = 16$	**(A1)**	
4(a)	$a = -4$	**B1**	
	$480 = \frac{180}{b}$ oe	**M1**	
	$b = \frac{3}{8}$	**A1**	
4(b)	Correct sketch  	**B2**	correct tan shape, two branches starting and finishing on same negative y value asymptote implied at $x = 240$ root between 120 and 240 **B1** for correct tan shape with exactly two branches plus one other correct property Maximum **B1** if not fully correct

Question	Answer	Marks	Partial Marks
5	$27x = (x^2)^2$ or $y = \left(\frac{y^2}{27}\right)^2$ oe	M1	if **M0** then, for first 4 marks, **SC4** if (3, 9) only stated and verified in both equations, ignore (0, 0) or **SC2** for (3, 9) only stated with no working, ignore (0, 0) If first **M1** then (3, 9) with no additional working award **M1SC1**
	$x^4 - 27x = 0$ or $y^4 - 729y = 0$ oe nfww	A1	
	$x(x^3 - 27) = 0$ or $y(y^3 - 729) = 0$ oe	M1	
	$A(3, 9)$ oe only nfww	A1	
	Mid-point = $(1.5, 4.5)$	B1	
	$m_{OA} = \frac{9}{3}$ oe	B1	
	$m_{\perp} = -\frac{3}{9}$ oe	M1	
	$y - 4.5 = -\frac{3}{9}(x - 1.5)$ oe isw	A1	**FT** *their* mid-point and *their* $-\frac{1}{\frac{9}{3}}$
6	$\frac{\mathrm{d}(\mathrm{e}^{\frac{x}{2}})}{\mathrm{d}x} = \frac{1}{2}\mathrm{e}^{\frac{x}{2}}$	B1	
	$\frac{\mathrm{d}(\cos 2x)}{\mathrm{d}x} = -2\sin 2x$ soi	B1	
	$x \times$ *their*$(-2\sin 2x) + \cos 2x$	M1	
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x} =\right] \frac{1}{2}\mathrm{e}^{\frac{x}{2}} - 2x\sin 2x + \cos 2x$	A1	**FT** *their* $\frac{\mathrm{d}\left(\mathrm{e}^{\frac{x}{2}}\right)}{\mathrm{d}x} = k\mathrm{e}^{\frac{x}{2}}$
	$\frac{\delta y}{h} = \textit{their} \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=1}$	M1	
	$-1.41[03...]h$ nfww	A1	

Question	Answer	Marks	Partial Marks
7	$4x^2 + kx + k - 2 = 2x + 1$	M1	
	$4x^2 + (k-2)x + k - 3\,[*0]$ soi	A1	* can be <, >, =, ≤, ≥
	$(k-2)^2 - 4(4)(k-3)$	M1	
	$k^2 - 20k + 52 * 0$	A1	
	$k = \dfrac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(52)}}{2}$	M1	
	$k = 10 \pm \sqrt{48}$ oe isw	A1	
	Alternative (using calculus) $2 = 8x + k$ oe	(M1)	
	$y = 4x^2 + (2-8x)x + 2 - 8x - 2$ or $y = -4x^2 - 6x$	(M1)	
	$0 = 4x^2 + 8x + 1$	(A1)	
	$x = \dfrac{-8 \pm \sqrt{8^2 - 4(4)(1)}}{8}$	(M1)	
	$x = -1 \pm \dfrac{\sqrt{48}}{8}$ oe	(A1)	
	for $k = 10 \pm \sqrt{48}$ oe	(A1)	
8(a)(i)	Valid explanation e.g. This ensures the argument of both logarithms is greater than 0	B1	
8(a)(ii)	$\log_a 6 = \log_a (y+3)^2$ oe	B1	
	$(y+3)^2 = 6$	M1	
	$y = -3 + \sqrt{6}$ oe only final answer	A1	

Question	Answer	Marks	Partial Marks
8(b)	Within a complete expression: Correct change of base to a: $\log_{\sqrt{b}} 9a = \frac{\log_a 9a}{\log_a \sqrt{b}}$ Correct use of power law: $\log_a \sqrt{b} = \frac{1}{2}\log_a b$ Correct use of addition/multiplication law: $\log_a 9a = \log_a 9 + \log_a a$ Correct use of $\log_a a = 1$	**M3**	**M2** for 2 or 3 correct steps within complete expression or **M1** for 1 correct step within complete expression
	leading to $2 + 3\log_a 9$. nfww	**A1**	
9	$\int \sin\left(6x - \frac{\pi}{2}\right)\mathrm{d}x = -\frac{\cos\left(6x - \frac{\pi}{2}\right)}{6} + c$	**B2**	**B1** for $\int \sin\left(6x - \frac{\pi}{2}\right)\mathrm{d}x = k\cos\left(6x - \frac{\pi}{2}\right) + c$ where $k < 0$ or $k = \frac{1}{6}$ or $-\frac{1}{6}\cos\left(6x - \frac{\pi}{2}\right)$
	$\frac{1}{2} = -\frac{1}{6}\cos\left(\frac{6\pi}{4} - \frac{\pi}{2}\right) + c$	**M1**	**FT** *their k* provided **B1** awarded
	$\int\left(-\frac{\cos\left(6x - \frac{\pi}{2}\right)}{6} + \frac{1}{3}\right)\mathrm{d}x$ $= -\frac{\sin\left(6x - \frac{\pi}{2}\right)}{36} + \frac{1}{3}x + A$	**M2**	**FT** *their k* $\cos\left(6x - \frac{\pi}{2}\right)$ + *their c* provided at least **B1** awarded **M1** for $m\sin\left(6x - \frac{\pi}{2}\right) + \left(\textit{their}\,\frac{1}{3}\right)x + A$ where $m < 0$ or $m = \frac{1}{36}$
	$\frac{13\pi}{12} = -\frac{1}{36}\sin\left(\frac{6\pi}{4} - \frac{\pi}{2}\right) + \frac{1}{3}\left(\frac{\pi}{4}\right) + A$	**M1**	**FT** *their m* and *their c* provided at least **M1** awarded
	$y = -\frac{\sin\left(6x - \frac{\pi}{2}\right)}{36} + \frac{1}{3}x + \pi$ oe cao	**A1**	

Question	Answer	Marks	Partial Marks
9	Alternative $\int -\cos 6x \, dx = -\frac{\sin 6x}{6} + c$	B2	**B1** for $\int -\cos 6x \, dx = k \sin 6x + c$ where $k < 0$ or $k = \frac{1}{6}$ or $-\frac{\sin 6x}{6}$
	$\frac{1}{2} = -\frac{1}{6}\sin\frac{3\pi}{2} + c$ oe	M1	**FT** *their k* provided **B1** awarded
	$\int\left(-\frac{\sin 6x}{6} + \frac{1}{3}\right)dx =$ $\frac{\cos 6x}{36} + \frac{1}{3}x + A$	M2	**FT** *their* $k \sin 6x +$ *their* c provided at least **B1** awarded **M1** for $m \cos 6x + \left(their \frac{1}{3}\right)x + A$ where $m > 0$ or $m = -\frac{1}{36}$
	$\frac{13\pi}{12} = \frac{\cos\frac{3\pi}{2}}{36} + \frac{1}{3}\left(\frac{\pi}{4}\right) + A$	M1	**FT** *their m* and *their c*
	$y = \frac{\cos 6x}{36} + \frac{1}{3}x + \pi$ oe cao	A1	
10(a)	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$	B1	
	$\sqrt{4^2 + 8^2}$	M1	**FT** *their* $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$
	$\frac{1}{\sqrt{80}}\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ oe isw	A1	**FT** provided working shown
10(b)	$\begin{pmatrix} 6 \\ -5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 10+x \\ 3+y \end{pmatrix}$ oe	M1	
	$x = 2,\ y = -13$	A1	

Question	Answer	Marks	Partial Marks
10(c)	$\overrightarrow{OE} = \frac{1}{1+\lambda}\begin{pmatrix}12\\7\end{pmatrix}$ oe seen	**B1**	
	Solves *their* $\frac{7}{1+\lambda} = 3$	**M1**	
	$\lambda = \frac{4}{3}$ oe	**A1**	
	Alternative $\overrightarrow{OE} = \begin{pmatrix}x\\3\end{pmatrix}$ $\frac{12}{7} = \frac{x}{3}$ $x = \frac{36}{7}$	**(B1)**	
	$\frac{1+\lambda}{1} = \frac{12}{36/7}$	**(M1)**	**FT** *their x*
	$\lambda = \frac{4}{3}$	**(A1)**	
11(a)(i)	$1 + d$, $1 + 7d$, $1 + 43d$ soi	**B1**	
	$[r =]$ *their* $\frac{1+7d}{1+d} =$ *their* $\frac{1+43d}{1+7d}$	**M2**	**FT** *their* ratios of terms provided in terms of a and d **M1 FT** for either $[r =]\frac{1+7d}{1+d}$ or $[r] = \frac{1+43d}{1+7d}$
	Simplifies to $6d^2 - 30d = 0$ oe nfww	**A1**	
	Verifies that $d = 5$ by substitution or factorises and solves to obtain $d = 5$ only	**A1**	

Question	Answer	Marks	Partial Marks
11(a)(i)	Alternative $1 + d$, $1 + 7d$, $1 + 43d$ soi	**B1**	
	$\left(\frac{7a-6}{a}\right)^2 = \frac{43a-42}{a}$ oe	**M2**	**M1** for $\frac{7a-6}{a}$ or for $\frac{43a-42}{a}$ oe
	$6a^2 - 42a + 36 = 0$ oe	**A1**	
	Finds $a = 6$ and uses it to show that $d = 5$ only	**A1**	
11(a)(ii)	$S_{20} = \frac{20}{2}\{2[1] + (20-1)(5)\}$	**M1**	
	970	**A1**	
11(b)(i)	7776 nfww	**B2**	**B1** for $6 \times 6^{5-1}$
11(b)(ii)	Valid explanation e.g. The sum to infinity does not exist for this GP as the common ratio is greater than 1.	**B1**	
12	x-coordinate of $A = 6$ soi	**B1**	
	x-coordinate of $B = 9$ soi	**B1**	
	$k - 3 = (9 - k)(k - 3)$	**M1**	
	$k = 8$ [therefore $C(8, 5)$]	**A1**	
	$(8-6)\times 5$ or 10 oe soi	**B1**	
	$\int_{their8}^{their9} (12x - 27 - x^2)\,dx$ $= \frac{12}{2}x^2 - 27x - \frac{x^3}{3}$	**M2**	**M1** for 2 correct terms
	*their*10 + F(*their* 9) – F(*their* 8)	**M1**	DEP on at least M1 for integration
	$\frac{38}{3}$ or $12\frac{2}{3}$ or 12.7 or 12.66[66...] rot to 4 or more figs nfww	**A1**	